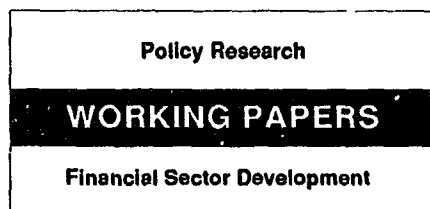


WPS 1145



Financial Sector Development Department  
The World Bank  
June 1993  
WPS 1145

# The Simple(r) Algebra of Pension Plans

Dimitri Vittas

Unless the real rate of interest exceeds the growth rate of real wages by a significant margin, payment of a reasonable pension rate requires a high contribution rate or a high active worklife ratio.

This paper — a product of the Financial Sector Development Department — is part of a larger effort in the department to study pension systems and contractual savings. Copies of the paper are available free from the World Bank, 1818 H Street NW, Washington, DC 20433. Please contact Priscilla Infante, room N9-003, extension 37664 (June 1993, 35 pages).

Chile's success with pension reform in the early 1980s and the continuing financial pressures facing the social security systems of many developing (and some industrial) countries have elicited considerable interest in the mechanics of pension systems that are based on fully funded individual capitalization accounts.

Vittas sets out the simple(r) algebra of both defined contribution and defined benefit plans. He notes the following results:

- Unless the real rate of interest exceeds the growth rate of real wages by a significant margin, payment of a reasonable pension rate requires a high contribution rate or a high active worklife ratio (the ratio of working life to retirement life).
- It is more difficult for a high-growth economy to provide a high pension rate, although the absolute level of the pension could be higher than the pension level in a low-growth economy with a high pension rate.
- When pensions are indexed to prices, the level of interest rates and growth rates affects positively the level of the pension rate. But when pensions are indexed to wages, only the difference between interest rates and growth rates has an effect on pension rates. In all cases, the active worklife ratio has a positive effect on the pension rate.
- The timing of contributions, and therefore the pattern of earnings over a person's full career, has a significant but not major effect on pension rates.
- Employer-based, final-salary, defined-benefit plans penalize early leavers and favor late high-fliers.
- Full funding with universal coverage and full lifetime careers would lead to a massive accumulation of capital, especially if the demographic structure resembles a pillar rather than the more traditional pyramid. This would have major implications for the productivity of capital and the functioning of financial systems.
- Full funding may not provide a solution to the financial pressures caused by demographic changes. An increase in the retirement age may be required under both funded and unfunded pension systems to accommodate population aging. This will result in a higher active worklife ratio and a lower dependency ratio.
- A mixed system — combining a redistributive first pillar and a fully funded defined-contribution second pillar — represents a more prudent, perhaps more equitable, approach to reforming pension systems.
- Variable contribution and pension rates, within specified limits, might be preferable and more consistent with greater reliance on personal choice.

The Policy Research Working Paper Series disseminates the findings of work under way in the Bank. An objective of the series is to get these findings out quickly, even if presentations are less than fully polished. The findings, interpretations, and conclusions in these papers do not necessarily represent official Bank policy.

**Financial Sector Department  
World Bank**

# **The Simple(r) Algebra of Pension Plans**

**Dimitri Vittas**

## I. INTRODUCTION<sup>1</sup>

The success of the Chilean pension reform of the early 1980s and the continuing financial pressures facing the social security systems of many developing and some developed countries have elicited considerable interest in the mechanics of pension systems that are based on fully-funded individual capitalization accounts. Such systems are known as defined-contribution systems because they pay pensions that depend on the contributions made during a person's working life and the investment income generated by investing the accumulated funds.

Fully-funded defined-contribution systems are one of three main forms of organizing the provision and financing of pensions. Many countries, both developed and developing ones, especially in Latin America, have social security systems that operate on a pay-as-you-go basis, i.e. pensions to retired people and their dependents are financed from the current contributions of active workers (in most cases, transfers from general tax revenue are also used to cover any deficit). Such systems are known as unfunded defined-benefit schemes because they do not accumulate any savings to fund future pensions and pension benefits are not directly linked to the contributions made on an individual basis. Pay-as-you-go systems face financial difficulties because of the aging of the population of many countries and also because of evasion, which not surprisingly is particularly pronounced in countries with high contribution rates.

The third main system of pension finance involves funded defined-benefit plans. The social security systems of some countries operate on a (partially) funded basis, while company-based pension schemes are also often based on funded defined-benefit systems.

The various types of pension schemes have their distinct operating characteristics and raise different economic and regulatory issues<sup>2</sup>. However, a common feature of funded pension plans, whether they are based on defined benefits or on defined contributions, is the accumulation of a capital fund over the working lives of members to provide a desired pension to its beneficiaries after retirement.

---

<sup>1</sup> The (r) in the title of this paper reflects the sad fact that pension issues are highly complex and almost nothing is really "simple" about them.

<sup>2</sup> Vittas and Skully (1992) provide an overview of the operating characteristics of different types of contractual savings institutions and pension systems, while James (1992) reviews policy issues in providing income security for old age.

A basic question facing policymakers in countries that contemplate the introduction of a pension system based on fully-funded individual capitalization accounts concerns the contribution rate that would be required to cover an adequate but affordable level of pension. Pension plan calculations can be complicated by a plethora of necessary actuarial assumptions. However, these actuarial assumptions somehow obscure a fundamentally simpler algebra that underlines the working of pension plans.

The purpose of this paper is to set out the relationship between contribution and pension rates under different assumptions about the length of working and retirement life and about real rates of interest and rates of growth of real wages<sup>3</sup>. In addition, the paper uses this algebra to illustrate some basic policy issues.

The paper is divided into four sections. Following this introduction, Section II sets out the simpler algebra of defined contribution plans. Section III develops the algebra of defined benefit plans. Finally, Section IV discusses a number of policy issues, such as the required contribution rates for defined contribution schemes, the treatment of early leavers and high fliers under defined benefit plans, the macrofinancial implications of funded schemes, and the contrast between funded and pay-as-you-go systems in dealing with demographic pressures,

Although the paper mostly deals with a mechanical application of the algebra involved, and makes no assumptions or predictions about behavioral changes or effects, a number of interesting results are worth noting:

- \* Unless the real rate of interest exceeds the growth rate of real wages by a significant margin of 2 to 3 percentage points, payment of a reasonable pension rate requires a high contribution rate and/or a high active worklife ratio<sup>4</sup>.
- \* It is more difficult for a high growth economy to provide a high pension rate, though the absolute level of the pension could be higher there than in a low growth economy with a high pension rate.
- \* When pensions are indexed to prices, the level of interest and growth rates affects positively the level of the pension rate. But when pensions are indexed to

---

<sup>3</sup> For other papers that examine the basic algebra of pension plans, see Bonilla (1992) and Schwarz (1992).

<sup>4</sup> I.e. the ratio of working life to retirement life.

wages, only the difference between interest rates and growth rates has an effect on pension rates. Of course, in all cases, the active worklife ratio has a positive effect on the pension rate.

- \* The timing of contributions, and therefore the pattern of earnings over a person's lifetime career, has a significant, though not major, effect on pension rates.
- \* Employer-based final salary defined benefit plans penalize early leavers (unless their pension rights are fully transferable) and favor late high fliers (who would in most corporations normally come from the ranks of senior management).
- \* Full funding with universal coverage and full lifetime careers would lead to a massive accumulation of capital, especially if the demographic structure resembles a pillar rather than the more traditional pyramid.
- \* A massive accumulation of pension fund assets would have far-reaching implications for the productivity of capital and the functioning of financial systems.
- \* Full funding may not provide a solution to the financial pressures caused by demographic changes. An increase in the retirement age may be required under both funded and unfunded pension systems in order to cope effectively with population aging. This will result in a higher active worklife ratio and a lower dependency ratio.
- \* In view of the considerable uncertainties involved, a mixed system combining a redistributive first pillar and a fully funded defined contribution second pillar would represent a more prudent, and perhaps also more equitable, approach in reforming pension systems.
- \* Although most of the analysis is based on constant contribution and pension rates, both rates could in practice be variable. In fact, allowing workers to set both their own contribution and pension rates, within a specified range, might be preferable and more consistent with greater reliance on personal choice.

## II. THE ALGEBRA OF DEFINED CONTRIBUTION PLANS

### 2.1 The simplest case

The simplest possible case is one where both the real rate of interest and the growth rate of real wages<sup>5</sup> are equal to zero. Pensions are assumed to be indexed to either prices or wages and thus inflation has no impact on the calculations involved (alternatively, inflation may be assumed to be zero)<sup>6</sup>.

The objective of the pension plan is to accumulate capital over the working life of workers which will be used to make pension payments over their retirement.

$$(1) \quad k * W * n = p * W * m$$

where

k is the contribution rate  
W is the gross real wage  
p is the gross pension (or replacement) rate, i.e. the ratio of the pension to the gross real wage  
n is the length of the working life and  
m is the length of life in retirement.

Solving equation (1) for the gross pension rate yields

$$(2) \quad p = k * \frac{n}{m}$$

The gross pension rate is greater, the greater the contribution rate and the greater the ratio of working life over retirement life. If the working life is 40 years and the retirement life is 20 years, and if the desired gross pension (or replacement) rate is 50%, the required contribution rate would be 25% of the gross wage<sup>7</sup>.

---

<sup>5</sup> It is the growth rate of the real wage of the individual worker that needs to be taken into account. This would reflect both the growth of average real wages and the progression of the individual worker in terms of seniority. However, to simplify matters the examples are based on average representative workers whose wages increase by the average growth rate of real wages.

<sup>6</sup> This paper assumes throughout that pensions are indexed either to prices or to wages. For a discussion of the effect of inflation when pensions are not indexed, see Schwarz (1992).

<sup>7</sup> It is interesting to note that in this case the capital fund accumulated at the time of retirement after 40 years of contributions would correspond to 10 times the annual gross wage.

Note, however, that "p" is the ratio of the pension to the gross wage before deducting contributions. If "b" is the net pension rate, i.e. the ratio of the pension to the net wage received after deducting contributions, then

$$(3) \quad b = \frac{p}{1-k}$$

Thus, with a 25% contribution rate, a 50% gross pension rate would correspond to a 66% net pension rate. To achieve a 100% net pension rate, a contribution rate of 33% would be required. The gross pension rate would then amount to 66%.

The following additional simple results can be obtained:

if  $k=10\%$ ,  $n=40$  and  $m=20$ , then  $p=20\%$  and  $b=22\%$   
 if  $k=10\%$ ,  $n=45$  and  $m=15$ , then  $p=30\%$  and  $b=33\%$   
 if  $k=10\%$ ,  $n=30$  and  $m=10$ , then  $p=30\%$  and  $b=33\%$   
 if  $k=20\%$ ,  $n=40$  and  $m=20$ , then  $p=40\%$  and  $b=50\%$   
 if  $k=20\%$ ,  $n=45$  and  $m=15$ , then  $p=60\%$  and  $b=75\%$ .

It can be seen that in the case where the real rate of interest and the growth rate of real wages are both equal to zero, a high contribution rate would be required in order to receive a "decent" pension, i.e. a pension that corresponds to a relatively high net replacement ratio. As discussed below, a high contribution rate would be required for a decent pension in all cases, except where the real rate of interest is much greater than the growth rate of the real wage, or the active worklife ratio is very high.

## 2.2 The case where the real rate of interest is positive

The algebra becomes somewhat more complex if we allow for a positive real rate of interest but continue to assume zero growth in real wages. If contributions are paid at the end of each year, the first year's contributions will accumulate interest for  $(n-1)$  years, those of the second year for  $(n-2)$  years, and so on. At the time of retirement, the accumulated value of the fund will be equal to the value of pension payments, which will be given by a series of discounted payments, with the last one discounted for "m" years. The objective function of the pension plan will then look as follows:

$$(4) \quad k * W * S = p * W * A$$

where

S is the accumulated value of a stream of payments for n years invested at a real interest rate of r per cent and



**A** is the discounted value of a stream of payments for **m** years at a real rate of interest of **r** per cent.

$$(5) \quad S = (1+r)^{(n-1)} + (1+r)^{(n-2)} + \dots + (1+r) + 1$$

and

$$(6) \quad A = \frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \dots + \frac{1}{(1+r)^m}$$

Equation (2) will then become

$$(7) \quad p = k * \frac{S}{A}$$

Note that the values of "**S**" and "**A**" will be equal to "**n**" and "**m**" respectively if the rate of interest is zero. At a positive rate of interest, "**S**" is greater than "**n**", while "**A**" is smaller than "**m**". Thus, for a given contribution rate, a higher pension rate is obtained.

With a real rate of interest of 3%, "**n**" equal to 40 and "**m**" equal to 20, "**S**" amounts to 75.40 and "**A**" to 14.88. A 10% contribution rate would then fund a gross pension rate of 51% and a net pension rate of 56%.

However, if retirement is delayed by 5 years, the working life would increase to 45 years and, assuming no change in life expectancy, the retirement life will decline to 15 years, giving an active worklife ratio of 3. Under these conditions and continuing to assume a 3% real rate of return, "**S**" will amount to 92.72 and "**A**" to 11.94. The gross pension rate from a 10% contribution rate would then equal 78%, corresponding to a net pension rate of 87%.

### 2.3 The case where the real interest rate and the growth rate of real wages are positive and equal, and pensions are indexed to price inflation

When we allow for a positive rate of growth of the real wage as well as a positive real rate of interest, the formula for the accumulation of the capital fund is more complicated. If the plan aims to provide a pension that is a fraction of the final salary, the objective function of the pension plan has to take

account of both the growth in wages and the level of the final wage<sup>8</sup>:

$$(8) \quad k * W * F = p * W * (1+g)^{(n-1)} * A$$

where

$F$  is a stream of payments that grow by "g" per cent and earn interest at "r" per cent, i.e.

$$(9) \quad F = (1+r)^{(n-1)} + (1+g)(1+r)^{(n-2)} + \\ + (1+g)^2(1+r)^{(n-3)} + \dots + \\ + (1+g)^{(n-2)}(1+r) + (1+g)^{(n-1)}$$

With  $g=r$ ,

$$(10) \quad F = n * (1+g)^{(n-1)}$$

and equation (8) becomes

$$(11) \quad k * W * n * (1+g)^{(n-1)} = p * W * (1+g)^{(n-1)} * A$$

and

$$(12) \quad p = k * \frac{n}{A}$$

With  $g=r>0\%$ , a given contribution rate produces a pension rate that is higher than when  $g=r=0\%$ , but lower than when  $r>0\%$  and  $g=0\%$ . Thus, with a contribution rate of 10%, "n" equal to 40 and "m" equal to 20, the gross pension rate when  $r=g=3\%$  will be 27% and the net pension rate 30%. With "n" equal to 45 and "m" equal to 15, the gross pension rate will be 38% and the net pension rate 42%.

#### 2.4 The case where "g" and "r" are positive but not equal and pensions are indexed to prices

If "g" and "r" are positive but not equal, the value of the fund at retirement is given by the sum of the geometric series of equation (9).

---

<sup>8</sup> If pension plans aim to provide a pension related to average actualized lifetime earnings, the right-hand side of equation (8) must be multiplied by the ratio of average wages to the final wage.

$$(13) \quad F = \frac{(1+r)^n - (1+g)^n}{(r - g)}$$

Equation (11) then becomes

$$(14) \quad k * W * F = p * W * G * A$$

where  $G = (1+g)^{(n-1)}$  and

$$(15) \quad p = k * \frac{F}{G * A}$$

For a given contribution rate, the pension rate depends on the relative values of "g" and "r" as well as the values of "n" and "m". The greater the growth rate of real wages, the higher the real rate of interest that is required to achieve a desired pension rate as a percentage of the final wage. However, a higher "g" also implies a higher final wage so that a lower pension rate may in fact correspond to a higher pension in absolute terms.

Tables 1a and 1b (and Figures 1a and 1b) show the relationship between the gross pension rate and the real interest rate for different growth rates of real wages, when pensions are indexed to prices. It can be seen from Table 1a that with  $k=10\%$ ,  $n=40$  and  $m=20$ , when  $g=2\%$  and  $r=3\%$  the gross pension rate amounts to 33%, but when  $g=3\%$  and  $r=2\%$  the gross pension rate falls to 20%. A 3 percentage points difference between the real rate of interest and the growth rate of real wages is required for a pension rate in excess of 50%.

Table 1b provides the same information for "n" equal to 45 and "m" equal to 15. It can be seen that when  $g=2\%$  and  $r=3\%$  the gross pension rate is 47%, while when  $g=3\%$  and  $r=2\%$  it is 29%. These pension rates are higher than those shown in Table 1a. They reflect the longer working and shorter retirement lives and the effects of compounding. A 2 percentage points difference between "r" and "g" is adequate to produce gross pension rates in excess of 50%. When  $r=5\%$  and  $g=3\%$ , the pension rate amounts to 68%.

When pensions are indexed to prices, the levels of interest rates and wage growth rates also affect the level of pension rates. Thus, as shown in Table 1a, when  $r=g=1\%$  the pension rate amounts to 22%, but rises to 27% with  $r=g=3\%$ . The corresponding pension rates in Table 1b are 33% and 38%.

In considering the entries of Tables 1a and 1b (as well as those of following tables) it should be noted that only those combinations that result in reasonable pension rates of between 50% and 70% would be compatible with the assumed contribution

Table 1a

Targeted Pension Rate with Pensions Indexed to Prices  
 $k=10\%$ ,  $n=40$ ,  $m=20$

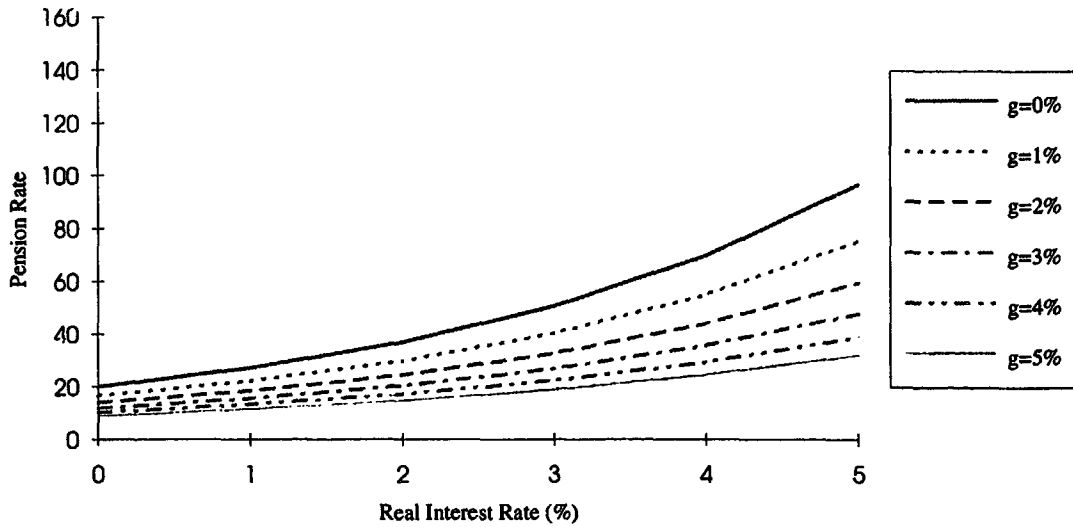
	Growth Rate of Real Wages					
	0%	1%	2%	3%	4%	5%
Real Interest Rate						
0%	20.0	16.6	14.0	11.9	10.3	9.0
1%	27.1	22.2	18.4	15.5	13.3	11.5
2%	36.9	29.8	24.5	20.4	17.2	14.7
3%	50.7	40.4	32.7	26.9	22.4	18.9
4%	69.9	55.1	44.1	35.8	29.4	24.6
5%	96.9	75.5	59.7	47.9	38.9	32.1
10%	519.9	387.5	292.0	222.5	171.6	133.9

Table 1b

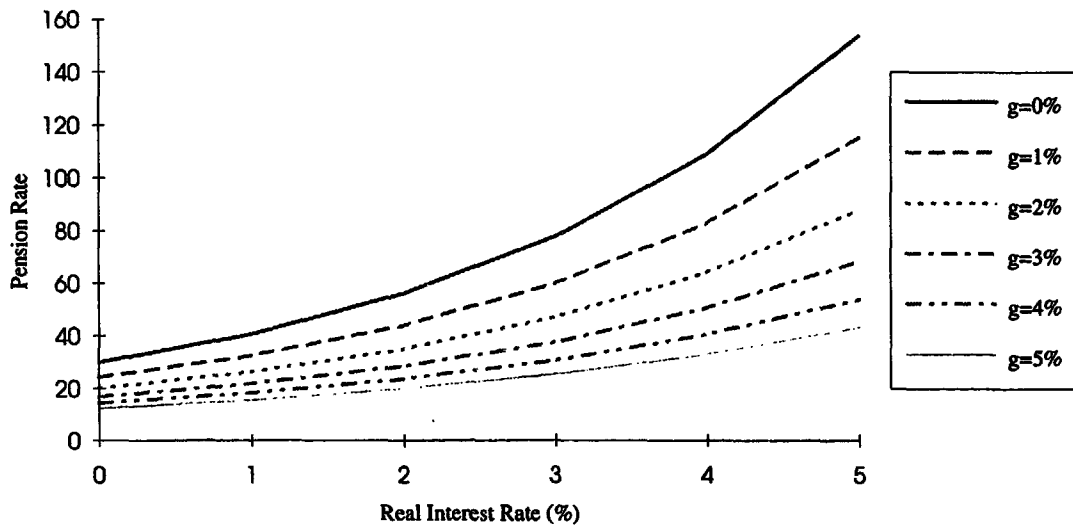
Targeted Pension Rate with Pensions Indexed to Prices  
 $k=10\%$ ,  $n=45$ ,  $m=15$

	Growth Rate of Real Wages					
	0%	1%	2%	3%	4%	5%
Real Interest Rate						
0%	30.0	24.3	20.0	16.8	14.4	12.4
1%	40.7	32.5	26.4	21.8	18.3	15.6
2%	56.0	43.9	35.0	28.5	23.6	19.9
3%	77.7	59.9	47.1	37.7	30.7	25.5
4%	108.9	82.8	64.0	50.5	40.5	33.0
5%	153.9	115.4	88.0	68.3	53.9	43.4
10%	945.2	672.5	484.4	353.5	261.6	196.4

**Figure 1a**  
**Targeted Pension Rate with Pensions Indexed to Prices**  
**(  $k=10\%$ ,  $n=40$ ,  $m=20$  )**



**Figure 1b**  
**Targeted Pension Rate with Pensions Indexed to Prices**  
**(  $k=10\%$ ,  $n=45$ ,  $m=15$  )**



rate and working and retirement lives. For instance in Table 1a, only the combinations  $r=3\%$  &  $g=0\%$ ,  $r=4\%$  &  $g=1\%$ ,  $r=5\%$  &  $g=2\%$ , and  $r=5\%$  &  $g=3\%$ , would produce reasonable pension rates. Entries to the top right of these combinations would imply a higher required contribution rate or worklife ratio for the achievement of reasonable pension targets, while entries to the bottom left would imply a lower required contribution rate or worklife ratio. Moreover, some combinations, such as those involving a 10% real rate of interest over a 60 year period, would be unlikely to occur.

## 2.5 The case of indexing pensions to wages with $g=r$

Indexing pensions to prices maintains their real value. However, if one is concerned to maintain the value of pensions relative to wages and thus ensure that pensioners benefit from continuing growth, pensions must be indexed to wages. Of course, if real wages fall, due to macroeconomic crisis, then real pensions would also decline. Indexing pensions to wages is generally more expensive and requires higher contributions rates, if real wage growth is positive.

When pensions are indexed to wages, the value of pension payments at the time of retirement is given by

$$(16) \quad D = \frac{(1+g)}{(1+r)} + \frac{(1+g)^2}{(1+r)^2} + \dots + \frac{(1+g)^{(m-1)}}{(1+r)^{(m-1)}} + \frac{(1+g)^m}{(1+r)^m}$$

If  $g=r$ , then equation (16) simplifies to

$$(17) \quad D = m$$

and equation (11) becomes

$$(18) \quad k * W * n * G = p * W * G * m$$

and

$$(19) \quad p = k * \frac{n}{m}$$

This is identical to equation (2) and provides a simple approximation to the required contribution rate when the real rate of interest and the growth rate of real wages are assumed to be equal. Since in real life, " $g$ " and " $r$ " are determined endogenously and cannot deviate much from each other for long periods of time, equation (19) (though very simple) offers a good approximation of the required contribution rate for a targeted pension rate that effectively depends on the active worklife ratio.

## 2.6 The case of indexing pensions to wages with unequal "g" and "r"

If "g" and "r" are not equal, then the value of wage-indexed pensions at the time of retirement is given by the sum of the geometric series of equation (16). This is equal to

$$(20) \quad D = \left[ \frac{(1+g)^m - 1}{(1+r)^m} \right] * \frac{(1+g)}{g-r}$$

In this case, equation (14) becomes

$$(21) \quad k * W * F = p * W * G * D$$

and equation (15)

$$(22) \quad p = k * \frac{F}{G * D}$$

Tables 2a and 2b (and Figures 2a and 2b) show the relationship between the gross pension rate and the real interest rate for different growth rates of real wages, when pensions are indexed to wages. In general, wage indexation of pensions, when real wages are rising, requires either a higher rate of return or a higher rate of contribution to achieve a given pension rate.

If  $g < r$ , "D" is smaller than "m" but greater than "A", so that, for a given contribution rate and when the growth rate of the real wage is assumed to be positive, wage indexation of pensions causes a reduction in pension rates. Table 2a shows that with  $k=10\%$ ,  $n=40$ ,  $m=20$ ,  $g=2\%$  and  $r=3\%$ , the gross pension rate will be 27%. This is lower than the pension rate of 33% achieved when pensions are indexed to prices.

If  $g > r$ , then "D" is greater than "m". A very high contribution rate would be required to achieve a decent pension rate. Thus, with  $g=3\%$  and  $r=2\%$  the pension rate amounts to only 15%. A contribution rate of 30% would be needed to achieve a gross pension rate of 45% that would then correspond to a net pension rate of 64%.

Table 2b shows the same relationships with  $k=10\%$ ,  $n=45$  and  $m=15$ . The longer working and shorter retirement lives result in higher pension rates as in the case of Table 1b.

When pensions are indexed to wages, the pension rate is determined by the difference between the real rate of interest and the growth rate of real wages. The levels of these two variables have little impact. Of course, the contribution rate and the length of the working and retirement lives continue as

basic determinants of pension rates. Thus, with  $k=10\%$ ,  $n=40$  and  $m=20$ , a difference of 2% between "r" and "g" produces a pension rate of 36-37%, while with  $k=10\%$ ,  $n=45$  and  $m=15$ , the pension rate is 55-56%.



Table 2a

Targeted Pension Rate with Pensions Indexed to Wages  
 $k=10\%$ ,  $n=40$ ,  $m=20$

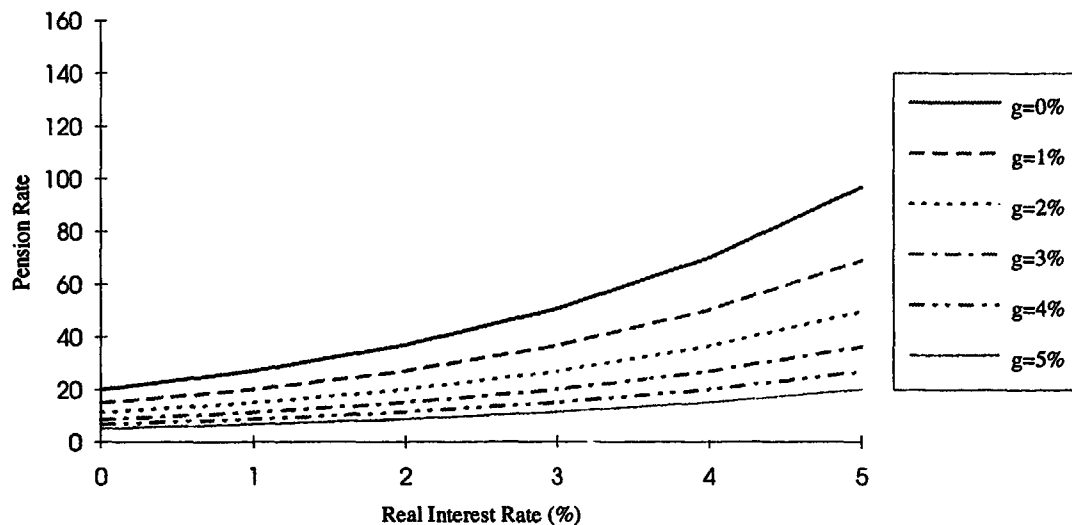
	Growth Rate of Real Wages					
	0%	1%	2%	3%	4%	5%
Real Interest Rate						
0%	20.0	14.9	11.3	8.6	6.7	5.2
1%	27.1	20.0	15.0	11.3	8.7	6.7
2%	36.9	27.0	20.0	15.0	11.4	8.7
3%	50.7	36.7	26.9	20.0	15.0	11.4
4%	69.9	50.2	36.5	26.9	20.0	15.1
5%	96.9	69.0	49.7	36.3	26.8	20.0
10%	519.9	359.1	250.3	176.0	125.0	89.6

Table 2b

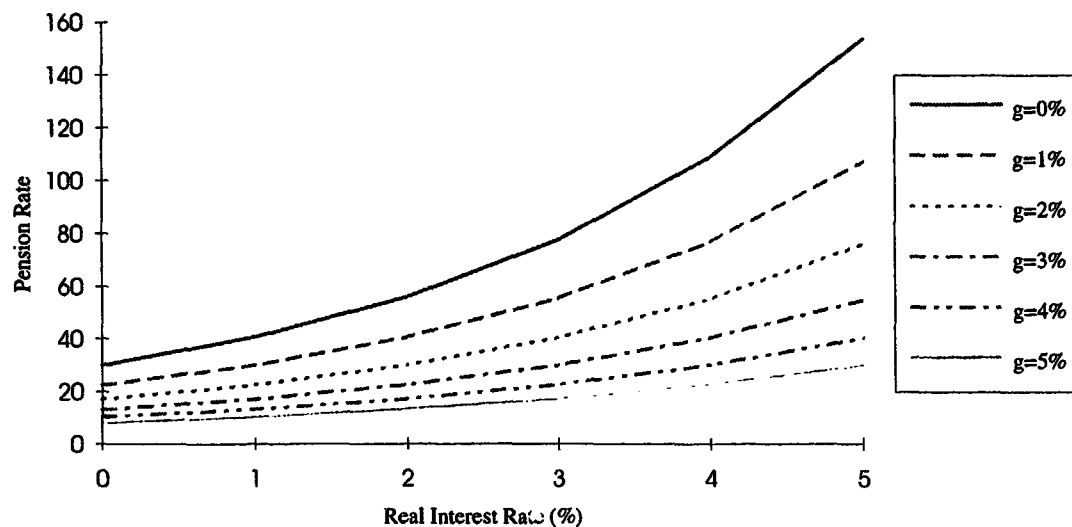
Targeted Pension Rate with Pensions Indexed to Wages  
 $k=10\%$ ,  $n=45$ ,  $m=15$

	Growth Rate of Real Wages					
	0%	1%	2%	3%	4%	5%
Real Interest Rate						
0%	30.0	22.4	17.1	13.2	10.4	8.2
1%	40.7	30.0	22.5	17.1	13.3	10.4
2%	56.0	40.6	30.0	22.6	17.2	13.4
3%	77.7	55.6	40.5	30.0	22.6	17.3
4%	108.9	76.9	55.3	40.4	30.0	22.7
5%	153.9	107.4	76.2	54.9	40.3	30.0
10%	945.2	631.3	426.4	291.5	201.8	141.6

**Figure 2a**  
**Targeted Pension Rate with Pensions Indexed to Wages**  
**(  $k=10\%$ ,  $n=40$ ,  $m=20$  )**



**Figure 2b**  
**Targeted Pension Rate with Pensions Indexed to Wages**  
**(  $k=10\%$ ,  $n=45$ ,  $m=15$  )**



### III. THE ALGEBRA OF DEFINED BENEFIT PLANS

This paper has so far dealt with plans based on constant contribution rates. These are mainly defined contribution plans, especially when they are established by the authorities and are mandatory on all employees. In such cases, a common contribution rate normally applies to all employees irrespective of their age and years of service, although the contribution rate may be adjusted from time to time.

However, there is no reason why defined contribution plans should not operate with variable rates or with rates that vary for different groups of employees<sup>9</sup>. For instance, in the Chilean pension scheme, the compulsory contribution rate is 10%, but affiliates can make additional voluntary contributions up to 20% of earnings (Vittas and Iglesias 1992). In Singapore, the standard contribution rate is a very high 40% of wages, but special lower rates apply for older workers.

Variable contribution rates are more extensively used for defined benefit plans, where the pension rate depends on the years of service of members. Although sponsoring firms make annual contributions on the basis of projections about the group structure of covered members in terms of age, salary level and length of service and in the light of actual and anticipated investment returns (Bodie 1990), in principle the collective contributions are based on contributions that can be calculated separately for each employee.

The algebra of pension plans with variable contribution rates is more complex than that for constant contribution rates. This is because a whole series of contribution rates need to be determined rather than the single rate used in plans with constant contribution rates.

#### 3.1 The case where $r=g=0$

As already noted, if  $g=r=0$ , the objective function of the pension plan is given by

---

<sup>9</sup> In systems with individual capitalization accounts, workers may be given the right to vary their annual contribution rates within an approved range, depending on whether their accumulated balances are adequate for meeting a targeted pension rate. It should also be noted that targeted pension rates need not be constant throughout a person's retirement. If consumption needs decline with age, a phased reduction in pension rates could be built into the plans. Such an approach may be preferable to offering unindexed pensions, the real value of which would decline in an unpredictable way.

$$(23) \quad k * W * n = p * W * m$$

In a defined benefit plan, the pension rate is equal to

$$(24) \quad p = n * q$$

where "q" is the rate of accrual of pension benefit as a per cent of salary per year of service. The contribution rate is then constant and given by

$$(25) \quad k = q * m$$

With  $q=1\%$ ,  $n=40$  and  $m=20$ , a contribution rate of 20% would be needed to pay a pension rate of 40%. A  $q=2\%$  would require a contribution rate of 40% and would pay a pension rate of 80%.

### 3.2 The case where "r" is positive but "g" is zero

If  $g=0$  but "r" is positive, and pensions are indexed to prices, the contribution rate will vary from year to year, since contributions will be accumulating interest until the time of retirement. The contribution rate for the first year will be

$$(26) \quad k_1 = \frac{q * A}{(1+r)^{(n-1)}}$$

where, as above, "A" is the value at the time of retirement of a stream of payments for "m" years at a real rate of interest of "r" per cent.

The contribution rate for the second year will be given by

$$(27) \quad k_2 = \frac{q * A}{(1+r)^{(n-2)}}$$

and so on until the last contribution rate will be given by

$$(28) \quad k_n = q * A$$

In general,

$$(29) \quad k_i = \frac{q * A}{(1+r)^{(n-i)}}$$

If  $r=3\%$ ,  $q=1\%$ ,  $n=40$  and  $m=20$ , then  $k_1=4.70\%$  and  $k_n=14.88\%$ . If  $r=5\%$ , then  $k_1=1.86\%$  and  $k_n=12.46\%$ . (Figure 3a shows the path of annual contribution rates.)

### 3.3 The case where both "r" and "g" are positive

If both "r" and "g" are positive, the calculation of annual contribution rates becomes more complex and depends on whether accumulated or projected benefits are taken into account. Accumulated benefits cover benefits already earned with regard to past service, while projected benefits also include benefits to be earned with regard to future service.

In the economic literature on pension funds there is considerable debate about the usefulness of actuarial assumptions for ascertaining the future liabilities of pension funds and about the extent of underfunding or overfunding of such liabilities (Bodie and Shoven, 1983). In practice, however, actuarial assumptions are required for both methods. Estimating accumulated benefits requires assumptions about the planned retirement age and the expected retirement life of beneficiaries, the growth (i.e. price or wage indexation) of pensions, and the discount rate. For projected benefits, further assumptions about future wage growth are also required.

On the basis of projected benefits, the series of contribution rates with pensions indexed to prices will be

$$(30) \quad k_1 = \frac{(1+g)^{(n-1)}}{(1+r)^{(n-1)}} * q * A$$

$$(31) \quad k_2 = \frac{(1+g)^{(n-2)}}{(1+r)^{(n-2)}} * q * A$$

and

$$(32) \quad k_n = q * A$$

In general,

$$(33) \quad k_i = \frac{(1+g)^{(n-i)}}{(1+r)^{(n-i)}} * q * A$$

However, if  $g=r$ , then the contribution rates are constant and equal to

$$(34) \quad k = q * A$$

With  $g=r=3\%$  and  $q=1\%$ ,  $n=40$  and  $m=20$ , the contribution rate is equal to 14.88%. With  $g=r=5\%$ , it is 12.46%. With  $g=5\%$  and  $r=3\%$ , contribution rates start with a high  $k_1$  of 31.55% and decline to a  $k_n$  of 14.88%. In contrast, with  $r=5\%$  and  $g=3\%$ ,  $k_1$  is a low 5.88% rising gradually to a  $k_n$  of 12.46%. (Figure 3b shows the path of annual contribution rates for four different combinations of interest and wage growth rates.)

If reserves are set aside for accumulated benefits only, the fund will need to cover the value of the accrued benefit at the time of retirement, without any allowance for projected benefits to be accrued. This is given by the product of the accrual factor and the value of annuity at the time of retirement. Allowing for the fact that reserves will be invested for  $(n-1)$  years, the first year's contribution rate will be equal to

$$(35) \quad k_1 = \frac{q * A}{(1+r)^{(n-1)}}$$

For the second year of service, the contribution rate will be expressed as a fraction of the new wage  $W*(1+g)$ . After two years of service, the value of the accrued benefit will be equal to twice the accrual factor times the value of the annuity at the time of retirement times the wage growth factor. Accumulated reserves will be invested for  $(n-2)$  years. However, the first year's reserves will have accumulated interest for one year so that the second's year contribution will be given by

$$(36) \quad k_2 = \frac{T_2}{(1+g)}$$

where

$$(37) \quad T_2 = \frac{2q * A * (1+g)}{(1+r)^{(n-2)}} - \frac{q * A}{(1+r)^{(n-2)}}$$

After three years of service, the contribution rate will be equal to

$$(38) \quad k_3 = \frac{T_3}{(1+g)^2}$$

where

$$(39) \quad T_3 = \frac{3q * A * (1+g)^2}{(1+r)^{(n-3)}} - \frac{2q * A * (1+g)}{(1+r)^{(n-3)}}$$

and after "n" years of service

$$(40) \quad k_n = \frac{T_n}{(1+g)^{(n-1)}}$$

where

$$(41) \quad T_n = n * q * A * (1+g)^{(n-1)} - (n-1) * q * A * (1+g)^{(n-2)}$$

In general,

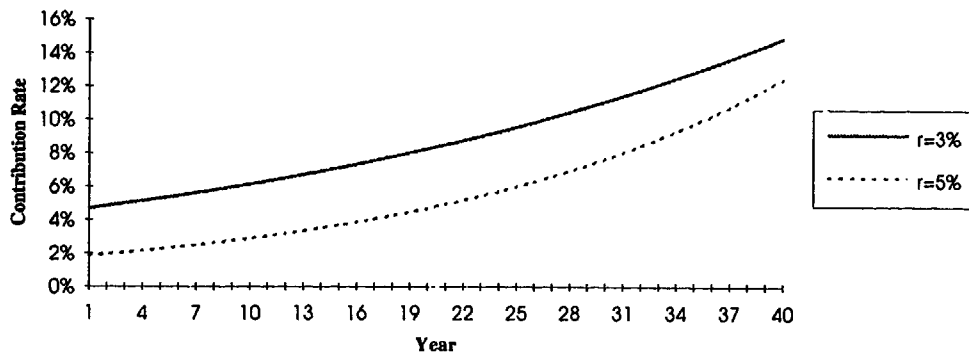
$$(42) \quad k_i = \frac{g * A * [i - (i-1)/(1+g)]}{(1+r)^{(a-i)}}$$

With  $g=3\%$  and  $r=5\%$ ,  $k_1 = 1.86\%$  and  $k_n = 26.62\%$ , but with  $g=5\%$  and  $r=3\%$ ,  $k_1 = 4.70\%$  and  $k_n = 42.51\%$ . (Figure 3c shows the path of annual contribution rates for the same four combinations of interest and wage growth rates as in Figure 3b.)

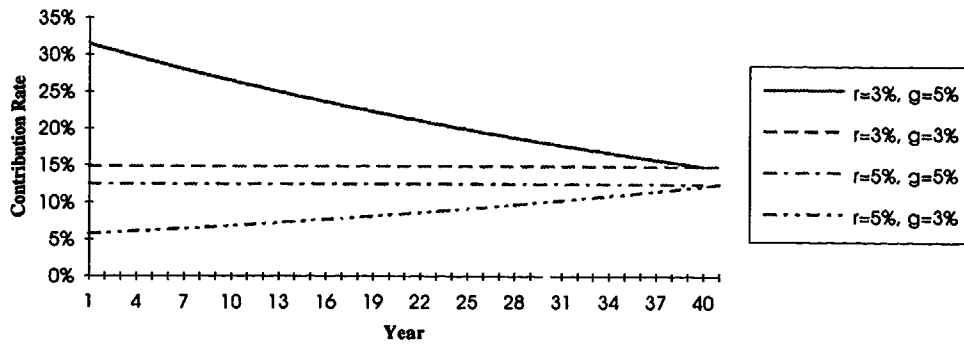
Using projected benefits for determining annual contribution rates smooths out the impact of future wage growth, but may require very large initial contribution rates if wage growth exceeds the rate of interest. On the other hand, using accumulated benefits as the basis for determining annual contribution rates may involve large changes in contribution rates from one year to the next as a result of wage growth and longer service. Contribution rates may need to increase steeply late in the working life of workers.

If pensions are indexed to wages, "A" must be replaced with "D" in equations (26) through (42). Since with a positive "g", "D" is greater than "A", wage indexation of pensions will require higher annual contribution rates under either method.

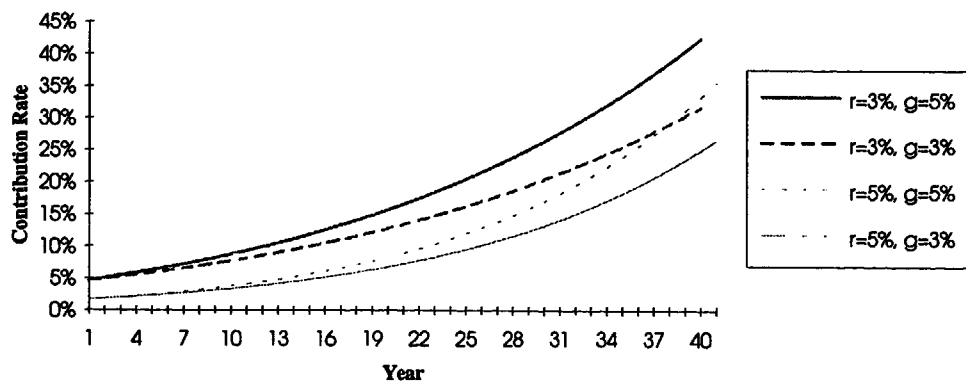
**Figure 3a**  
**Required Contribution Rate with Pensions Indexed to Prices**  
 (  $g=0\%$ ,  $q=1\%$ ,  $n=40$ ,  $m=20$  )



**Figure 3b**  
**Required Contribution Rate with Pensions Indexed to Prices**  
 ( Projected Benefit Basis:  $q=1\%$ ,  $n=40$ ,  $m=20$  )



**Figure 3c**  
**Required Contribution Rate with Pensions Indexed to Prices**  
 (Accumulated Benefit Basis:  $q=1\%$ ,  $n=40$ ,  $m=20$ )





#### IV. POLICY ISSUES

The algebra developed in this paper is mechanical in nature and does not purport to shed any light on the effects of changes in behavior arising from the creation of funded and unfunded pension schemes. Nevertheless, it can be used to highlight a number of important policy issues, such as the required contribution rates for defined contribution schemes, the treatment of early leavers and high fliers under defined benefit plans, the macrofinancial implications of funded schemes, and the contrast between funded and pay-as-you-go systems in dealing with demographic pressures.

##### 4.1 Required contribution rates

It is clear from Section II that a high targeted pension rate requires a high contribution rate unless the real rate of interest is substantially higher than the rate of growth of real wages.

Mandatory retirement savings schemes, which include both national provident funds and government mandated but privately run systems, such as the Chilean AFP system, impose contribution rates that vary over a wide range. In Singapore, the contribution rate has been in excess of 35% for most of the period since 1968. The long-term target of the Singaporean authorities is for a 40% contribution rate, divided equally between employers and employees. This nominal rate corresponds to an effective contribution rate of 33% (i.e. total contributions of 40 out of a total payroll cost of 120). In Malaysia, Nepal and Sri Lanka the combined nominal contribution rate amounts to 20% (raised to 22% in Malaysia in 1992), while in a number of African countries it is only 10%.

In Singapore, real wages have grown by more than 4% over the past three decades or so, while the real return credited on CPF balances has on average been around 2% (Vittas 1993). Thus, on the basis of Tables 1a and 1b, Singaporean workers on average wages would save enough over their working lives to buy an annuity that could pay a pension rate ranging between 46% (if they worked for 40 years and lived in retirement for another 20) and 62% (if they worked for 45 years and lived in retirement for another 15). If the pension is related to the net wage, then the net replacement rate would be 58% and 78% respectively. One could argue therefore that the high contribution rate in Singapore is justified by the high growth rate of real wages. However, the real rate of return has been low, while the right given to CPF members to withdraw their funds for investments in housing, approved securities, and other purposes (e.g. education) suggests that the CPF is not used to meet solely pension

objectives. The balances left on the individual accounts on retirement would effectively attain an even lower pension rate<sup>10</sup>.

The Chilean AFP pension system imposes a 10% compulsory contribution rate, which is entirely paid by employees. To attain a pension rate of 70% of final wages, which is apparently an objective of the scheme, would require a high real rate of return, a low growth rate of real wages or a high active worklife ratio. Table 1a shows that, with  $n=40$  and  $m=20$ , a real rate of interest of 5% would attain a pension rate of 60% if  $g=2\%$  and 48% if  $g=3\%$ . Table 1b shows that, with  $n=45$  and  $m=15$ , the pension rates would rise to 88% and 68%, respectively.

Proponents of the Chilean scheme also argue that the targeted pension rates would be achieved with lower real rates of return if account is taken of the fact that workers experience a higher rate of wage growth early in their career with a substantial deceleration as they grow older. This is because workers will accumulate a proportionally larger balance early on that will be available for interest compounding over a longer period. Assuming that wages increase by 3.06% a year over the first 20 years of a person's career and then by 1% over the next 20 years, so that at the end of 40 years the final wage will be the same as with a constant 2% growth rate, a 10% contribution rate will achieve a 63% pension rate if the real rate of interest is 5% throughout. This compares with 59% under a constant 2% real growth rate. The pension rate will be even higher if one assumes a steeper rise early on in a worker's career with a more dramatic deceleration later. If  $g_1$  is 4% and  $g_2$  is 0.15% (so that overall growth is still 2%), the pension rate will increase to 69%. Thus, it is the relationship of the rate of interest and the wage growth rate over time that is important<sup>11</sup>. The effect of uneven or nonlinear earnings profiles is significant, though not as profound as is sometimes argued.

---

<sup>10</sup> For most of its existence, the CPF has allowed withdrawals of accumulated balances in one lump sum at age 55. In recent years, however, a minimum balance under the so-called Minimum Sum Scheme must be kept on the account to buy a pension annuity starting at age 60. The minimum amount is fixed in absolute terms, is indexed to inflation, and appears to be adequate to buy a pension of around 25% of average wages.

<sup>11</sup> The objective function of the pension plan is

$$k * F_1 * (1+r)^{(a-v)} + k * F_2 * (1+g_1)^{(v-1)} = p * (1+g_1)^{(v-1)} * (1+g_2)^{(a-v)} * A$$

where  $F_1$  and  $F_2$  are the sinking funds accumulated in the first and second periods and "v" is the length of the first period.

## 4.2 The treatment of early leavers and high fliers

Defined benefit plans have a significant advantage over defined contribution plans in that they protect workers from investment risk and, if benefits are indexed, from inflation risk. However, realization of these advantages depends on continuing employment with the same employer and on the solvency and integrity of plan sponsors.

Defined benefit final salary plans suffer from two important disadvantages. These are the unequal treatment of early leavers and long stayers, on the one hand, and high fliers and slow plodders, on the other.

The problem of early leavers is more pronounced when leaving employees are denied any vested rights, or if they have vested rights but they are not allowed to transfer them to their new employer. If the annual accrual factor is 1.5% and average wage growth is 2%, employees who change jobs in midcareer would be entitled to two pensions on retirement, one from their first employer based on 30% of their salary at the time of leaving and one from the second employer based on 30% of their salary on retirement. With a 2% growth rate, a starting salary of \$10,000 would amount after 20 years to \$14,600, while after another 20 years it would be equal to \$21,600. Workers who change jobs in midcareer would receive a pension equal to 50% of their final salary. In contrast, long stayers with a similar salary profile would get 60% of their final salary. The losses suffered by early leavers would be even greater if pension rights are not protected from inflation. With a 5% inflation, the effective pension rate for early leavers would fall to 38%.

The problem caused by high fliers can be seen by considering equation (40) which gives the required contribution rate in the last year of service. With  $g=3\%$  and  $r=5\%$ , the required contribution for the last year would be 27%. However, if high fliers are assumed to get a 10% increase in salary in their last active year against a 3% average up to that time, equation (40) shows that their contribution rate should rise in the last year to 57%. A 30% salary rise in the last year would require a 125% contribution rate for that year.

In a defined contribution system, such additional contributions would have to be made explicitly and would be more transparent. But in defined benefit schemes, the effects of salary rises and promotion near the end of a person's career are obscured by the average contribution rates made on behalf of all employees. The unequal treatment between high fliers and slow plodders (or the average worker) is even greater when, as is usually the case, most workers experience a significant decline in the growth rate of their earnings late in their careers. It

is argued by some that the popularity of defined benefit company pension schemes is due to a significant extent to the favorable treatment of senior managers who get big promotions and large salary rises late in their careers.

A solution to the unequal treatment of early leavers and high fliers would involve regulations that mandate minimum vesting periods, full portability of vested rights, and use of average actualized lifetime earnings as the basis for determining pensions.

#### **4.3 Macroeconomic and financial implications of funded schemes**

Funded schemes accumulate long-term savings that are available for investment in long-term instruments, such as government bonds, mortgage bonds and corporate securities as well as real assets. A question of major macroeconomic and financial significance regards the pace and especially the limit of such accumulation in relation to a country's GNP.

To calculate the total size of accumulated funds, it is necessary to make assumptions about the income and age distribution of active and passive workers as well as about the coverage of the pension schemes and the career patterns of workers.

Regarding the income distribution, one assumption is to postulate that all workers start with the same income, which grows by a given percentage each year. The income distribution would then reflect the age distribution and the growth rate of wages. An alternative assumption is to postulate that all workers earn the same income, which increases over time by the given growth rate. The income distribution would then be completely flat.

Regarding the age distribution, one formulation is to assume a pillar population structure which would imply a zero population growth. The alternative is to use a pyramidal demographic structure associated with a positive population growth rate.

In mechanical terms, the limit to the accumulation of capital in relation to GNP is given by the algebra set out above. Taking the simple case where  $r=g=0$ , it is easy to see that if  $k=10\%$ ,  $n=40$  and  $m=20$ , workers with 40 years of contributions would accumulate funds equal to 4 times their annual income, workers with 39 years would have funds equal to 3.9 times their incomes, and so on. Thus, if it is assumed that the income distribution is flat, the population structure resembles a pillar, coverage is universal, and all workers have full careers, the total funds of pension plans for active workers would amount to 205% of labor income.

In addition to the balances of active workers, account must also be taken of the balances of pensioners or passive workers. Continuing to assume a pillar demographic structure and  $r=g=0$ , it can be seen that, if the pension rate is equal to 20% of wages (which it would have to be under these assumptions), the balances of pensioners would range from 3.8 times the annual wage for pensioners who are retired for one year, to 3.6 times for those who are retired for two years, and so on. The total funds of pensioners would then amount to 95% of annual wages.

Thus, under these assumptions, the balances of both groups would amount to 300% of the wage bill. Table 3a shows the size of balances for different combinations of wage growth rates and interest rates under the assumption of pensions indexed to wages, a constant contribution rate of 10%, working and retirement lives of 40 and 20 years respectively, and zero population growth (i.e. a pillar demographic structure). Table 3b shows the size of balances when the population growth rate is 1.6% and the resulting population structure is a pyramid<sup>12</sup>. (Figures 4a and 4b show the total size of accumulated funds as a proportion of the wage bill for different combinations of interest and wage growth rates.)

These calculations correspond to those of table 2a where higher interest rates imply higher pension rates and higher growth rates imply lower pension rates. However, it is easy to compute that, for a given targeted pension rate, a positive " $r$ " would imply lower accumulated balances, while a positive " $g$ " would imply the opposite. It is also easy to see that, again for a given pension rate, a longer working life and shorter retirement life would imply smaller accumulated funds, mainly because the required funds for a shorter retirement life would be smaller.

Table 2a shows that with pensions indexed to wages, a working life of 40 years, a retirement life of 20 years, a real wage growth rate of 2% and a real interest rate of 5%, a 10% contribution rate would provide a pension rate of 50%. Table 3a then shows that with a pillar demographic structure, universal coverage and full working careers, the total accumulated balances would be equal to just over 500% of the total wage bill<sup>13</sup>.

---

<sup>12</sup> It can be seen from Tables 3a and 3b that if  $r=g$  and pensions are indexed to wages, the same results obtain as when  $r=g=0$ .

<sup>13</sup> Table 3b shows that with a population growth rate of 1.6% and the resulting pyramidal demographic structure, total accumulated balances would correspond to just over 300% of the total wage bill.

Table 3a

**Accumulated Pension Fund Balances\***  
(per cent of total wage bill)

Real Interest Rate	Growth Rate of Real Wages					
	0%	1%	2%	3%	4%	5%
0%	300	257	223	196	174	156
1%	355	300	257	224	196	174
2%	423	354	300	258	224	197
3%	511	422	353	300	258	225
4%	624	508	420	353	300	258
5%	769	619	506	419	352	300
10%	2499	1903	1468	1148	910	731

\* Pensions indexed to wages,  $k=10\%$ ,  $n=40$ ,  $m=20$ , pillar demographic structure (0% population growth), dependency rate 50%.

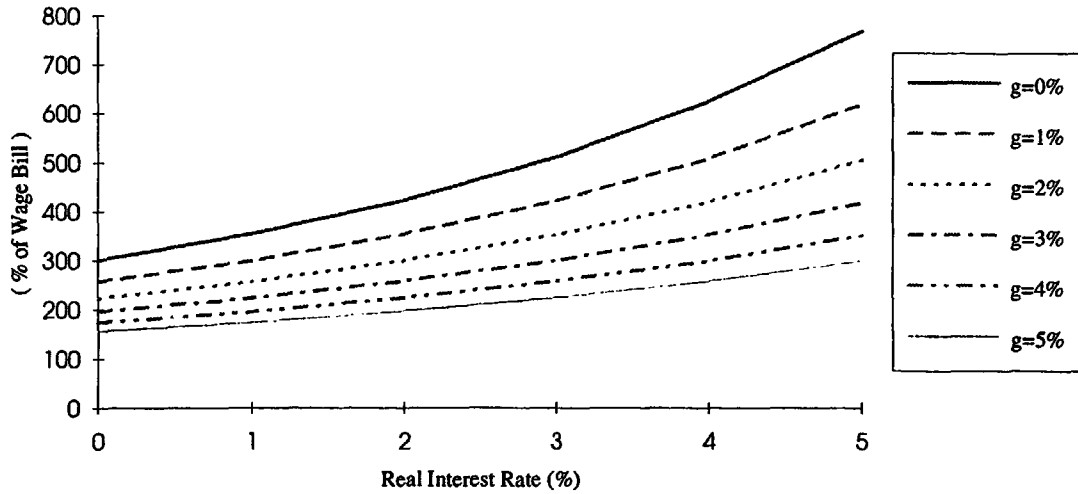
Table 3b

**Accumulated Pension Fund Balances\***  
(per cent of total wage bill)

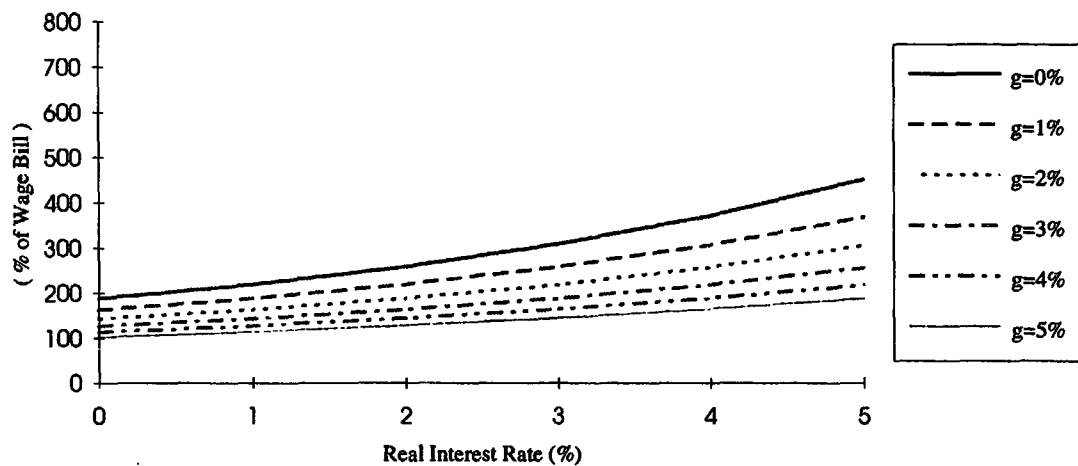
Real Interest Rate	Growth Rate of Real Wages					
	0%	1%	2%	3%	4%	5%
0%	188	163	143	127	114	103
1%	219	188	163	143	127	114
2%	258	219	188	163	144	128
3%	308	258	218	188	164	144
4%	371	306	257	218	188	164
5%	451	368	305	256	218	188
10%	1383	1065	831	658	528	430

\* Pensions indexed to wages,  $k=10\%$ ,  $n=40$ ,  $m=20$ , pyramidal demographic structure (1.6% population growth), dependency rate 30%.

**Figure 4a**  
**Accumulated Pension Fund Balances**  
**Pillar Population Structure, Pensions Indexed to Wages**  
**(  $k=10\%$ ,  $n=40$ ,  $m=20$  )**



**Figure 4b**  
**Accumulated Pension Fund Balances**  
**Pyramidal Population Structure (population growth 1.6%), Pensions**  
**Indexed to Wages**  
**(  $k=10\%$ ,  $n=45$ ,  $m=15$  )**



Assuming a labor income share of 70%, pension fund assets would amount to 350% of GNP. A smaller labor share, say 40%, would still imply total pension fund assets of 200% of GNP.

Such an increase in capital funds would imply an increase in the rate of saving, at least until the funds reach maturity. Historically, the adoption of funded pension schemes has not been associated with an increase in the rate of household and national saving, probably because of offsetting changes in household behavior. But the absence of a clear effect on the rate of saving may also be explained by the lack of universal coverage, the interrupted careers of many workers, the absence of full funding, the relative youth of most funded pension schemes, the low level of pension benefits (relatively few workers are covered by pension funds that provide a pension rate equal to 50% and indexed to wages), the low level of real returns (it is unclear whether a 5% real rate of return would prevail over a 60-year period in a country with universal coverage) and the pyramidal demographic structure (instead of the pillar structure assumed here). A forced saving scheme that does not suffer from these shortcomings may more than offset the reduction in voluntary saving.

These factors may also explain why the total assets of pension funds in different countries are nowhere near the very high levels quoted above. The highest level of assets in relation to GNP of pension funds and life insurance companies combined is found in Switzerland, the Netherlands and the United Kingdom where they range between 120% and 150% of GNP (Vittas and Skully 1991)<sup>14</sup>. In Singapore, where the CPF has been in existence since 1955 and where contribution rates are quite high, the total assets of the CPF correspond to around 70% of GNP. In some countries, total financial assets, including assets of banks and other deposit institutions, exceed 300%, and even 400% of GNP, but these totals include a significant amount of double counting.

The accumulation of large pension fund balances would also imply a domination of the financial system by pension funds. This would have far-reaching implications for the management, regulation and supervision of their assets and for the role of pension fund managers in corporate governance. Failure to properly regulate and supervise pension funds and ensure that they are invested safely and profitably would undermine their ability to pay the targeted pensions.

---

<sup>14</sup> Davis (1992) has data showing the rapid growth of total pension fund and life insurance assets over the past two decades or so.



Would pension fund assets ever attain such high levels? This would depend on their coverage, the career patterns of workers and the demographic structure as well as the relationship between the growth rate of real wages, the real rate of interest, and the age and wage/pension distribution of workers and pensioners. But to absorb the vast expansion in the availability of long-term savings implied by Tables 3a and 3b, capital intensity would have to increase substantially and would require significant technological advances in order to avoid a reduction in the productivity of capital. Otherwise, the implied reduction in real rates of return would question the feasibility of funding high levels of pension provision.

This question about the sufficiency of capital productivity and investment returns is often expressed as a concern about the adequacy of productive assets as collateral in which pension plans can invest their funds. Manufacturing assets are usually a small fraction of GNP, no higher than 40% to 50%. However, other potential assets include inventory, infrastructure, houses and land. Financial claims on these assets may be represented by mortgage and government bonds as well as corporate securities, in which accumulated funds may be invested. As already noted, there are several countries (e.g. Switzerland, the Netherlands and the United Kingdom) where pension funds and insurance companies already hold assets that correspond to well over 100% of GNP. Nevertheless, accumulation of pension fund balances on the scale suggested above would exert a downward pressure on real returns unless technological progress prevented a reduction in the productivity of capital.

Another, closely related, issue concerns the growing reliance of pension funds on capital gains for achieving their target rates of return. This has adverse implications for the stability and even sustainability of real returns and about the long-term feasibility of funded pension schemes with universal coverage. Evidence from OECD countries with relatively large funded pension sectors shows considerable variation in real rates of return over prolonged periods, ranging from negative levels between 1966 and 1980 to strongly positive ones over the 1980s (Table 4).

Thus, unless economic productivity were to increase substantially as a result of the increased availability of long-term savings, a fully funded pension system with universal coverage may not be able to pay targeted pensions.

Table 4

## Real Pension Fund Returns

	US \	UK	Netherlands	Switzerland
1996-70	-5.4	4.2	1.7	0.8
1971-75	-0.8	-2.8	-1.4	-0.5
1976-80	-1.9	4.9	2.0	4.0
1981-85	8.1	12.4	10.5	3.0
1986-1990	11.2	10.1	6.3	-0.2
1966-90	2.2	5.8	4.0	1.5

Source: Davis (1992)

#### 4.4 Demographic pressures and funded and pay-as-you-go schemes

The question of the adequacy of capital productivity and investment returns for fully funded schemes with universal coverage is also related to the issue of whether funded schemes can cope better than pay-as-you-go schemes with aging populations and the resulting demographic pressures. As shown in Tables 3a and 3b, a pillar demographic structure (zero population growth) implies a higher level of pension fund assets than a pyramidal structure.

This issue can be addressed by comparing the simple algebra of defined contribution plans with that of pay-as-you-go schemes. The required contribution rate for a funded scheme, when pensions are indexed to wages, is given by

$$(43) \quad c = p * \frac{G * D}{F}$$

where "G", "D" and "F" are defined as above. If "g" is equal to "r", equation (43) simplifies to

$$(44) \quad c = p * \frac{G * m}{G * n} = p * \frac{m}{n}$$

where the fraction  $m/n$  can be seen as the passivity ratio, i.e. the ratio of the retirement life over the working life<sup>15</sup>. The level of this ratio will depend on the age at which workers start

---

<sup>15</sup> The passivity ratio is the inverse of the active worklife ratio. The passivity ratio corresponds to the dependency ratio in social security parlance, while its inverse, the active worklife ratio, corresponds to the support ratio.

to contribute to a pension fund, their retirement age, and their expected retirement life.

For pay-as-you-go schemes, the required contribution rate when pensions are indexed to wages is given by

$$(45) \quad c = p * \frac{B}{C} = p * d$$

where "B" is the number of beneficiaries, "C" the number of contributors, and "d" the dependency ratio.

The value of the dependency ratio is determined by the same factors as the passivity ratio, except that it is also affected by the shape of the demographic structure. In a society with a demographic structure that resembles a pillar, the two ratios would have the same value. If people work for 40 years and live in retirement for 20 years, the dependency ratio would be equal to 50%, the same as the passivity ratio. If working life is extended to 45 years and retirement life is reduced to 15, both ratios would fall to 33%.

Thus, if the rate of interest and the growth rate of real wages are equal, funded and pay-as-you-go systems would require the same contribution rate if the demographic structure resembles a pillar. Pay-as-you-go systems would require a lower contribution rate if the demographic structure resembles a pyramid and a higher contribution rate if it resembles an inverted pyramid<sup>16</sup>. However, the same pattern could also apply to funded schemes.

This is so because the relationship between real interest rates and real wage growth rates would also be affected by the shape of the demographic structure. An economy with a young population and a low dependency ratio would tend to be characterized by a relative scarcity of capital. In such an economy, the real rate of interest would tend to be high, while the growth rate of real wages would tend to be depressed. A low contribution rate would then be needed for funded schemes as in

---

<sup>16</sup> Historically, demographic structures resembled pyramids with a large base. This translated into low dependency rates so that a given pension rate could be achieved with a low contribution rate. For instance, with a dependency ratio of 10%, a pension rate of 80% required a contribution rate of only 8%. But with the aging of the population, the demographic structure changes shape, first to a pyramid with a smaller base, and then to a pillar, requiring a substantial rise in contribution rates to achieve the same pension rate.

the case of pay-as-you-go schemes. With the aging of the population and increase in the dependency ratio, there would be a shift towards a relative scarcity of labor, which would tend to produce a higher growth rate of real wages and a lower real interest rate, causing a rise in the required contribution rate.

The implication of this analysis is that funding may not eliminate intergenerational transfers when demographic structures change, although a lot will depend on what happens to economic productivity and the real rate of interest when the capital intensity of production increases. In response to demographic pressures from population aging, the retirement age may need to be raised, regardless of whether pensions are funded or not. This will result in a higher active worklife ratio (a lower passivity ratio) and therefore a lower dependency ratio.

#### **4.5 Advantages of a Multi-Pillar Structure**

The uncertainties regarding investment returns, capital productivity, demographic trends, not to mention political and economic stability, suggest that an approach based on a multi-pillar system may be preferable on economic, financial and social grounds<sup>17</sup>. A multi-pillar approach would combine a pay-as-you-go (or partially funded) first pillar offering a pension equal to, say, 30% of net wages with a fully funded second pillar based on individual capitalization accounts offering an additional 40% of net wages. Such a system may combine the main advantages of the pay-as-you-go and funded systems, while avoiding the disadvantages of extreme solutions.

The first pillar could be redistributive, replacing a higher proportion of income for low income workers and a lower proportion for high income workers<sup>18</sup>. It could consist of two parts, one offering a flat pension and the other an earnings-related pension. The contribution rate for the first pillar, assuming it was financed from payroll taxes, would depend on the demographic structure (and the level of any partial pre-funding). For countries with mature populations, a contribution rate of 10% for the first pillar would be adequate for paying a 30% net pension rate. But for countries with younger populations and lower dependency ratios, the contribution rate for the first pillar could be significantly lower.

---

<sup>17</sup> A proposed strategy for pension reform based on a multi-pillar structure is set out in Vittas (1993).

<sup>18</sup> A redistributive system, funded from payroll or general taxes, would be needed to cover workers with limited and interrupted careers as well as their dependents. This is a major issue in all countries, including both Chile and Singapore.

The second pillar would require a contribution rate of 10% to 15%, depending on what assumptions are made about the future combination of interest rates and wage growth. The net pension rate of 40% that might be targeted in the second pillar would be easy to achieve with the suggested contribution rates as can be verified from Tables 2a and 2b. Thus, the total contribution rate could vary from 15% in the case of young countries to 25% in the case of old ones.

Before concluding this paper it is worth repeating the point made in footnote 9 above. While most of the analysis is based on constant contribution and pension rates, both rates could in practice be variable. In fact, allowing workers to set both their own contribution and pension rates, within a specified range, might be preferable and more consistent with greater reliance on personal choice.

## References

- Bodie, Zvi (1990). "Pensions as Retirement Income Insurance", *Journal of Economic Literature*, March.
- Bodie, Zvi and John Shoven (ed) (1983). *Financial Aspects of the United States Pension System*, Chicago: University of Chicago Press/NBER.
- Bonilla, Alejandro (1992). *Matematicas Financieras y Actuariales Minimas Para el Analisis de Fondos Individuales de Pensiones*, ILO, Department of Social Security, Geneva, mimeo.
- Davis, E. Philip (1992). *The Structure, Regulation and Performance of Pension Funds in Nine Industrial Countries*, World Bank, CECFP, mimeo, November.
- James, Estelle (1992). *Income Security for Old Age*, World Bank, Policy Research Working Papers, WPS 977, September.
- Vittas, Dimitri (1993). *Swiss Chilanpore: The Way Forward For Pension Reform?*, World Bank Policy Research Working Papers, WPS 1093, February.
- Vittas, Dimitri and Michael Skully (1991). *Overview of Contractual Savings Institutions*, World Bank, PRE Working Papers, WPS 605, March.
- Vittas, Dimitri and Augusto Iglesias (1992). *The Rationale and Performance of Personal Pension Plans in Chile*, World Bank, PRE Working Papers, WPS 867, February.
- Schwarz, Anita (1992). *Basic Parameters of Mandatory Retirement Savings Schemes*, World Bank, CECFP, mimeo, October.

# Policy Research Working Paper Series

Title	Author	Date	Contact for paper
WPS1123 Social Development is Economic Development	Nancy Birdsall	April 1993	S. Rothschild 37460
WPS1124 A New Database on Human Capital Stock: Sources, Methodology, and Results	Vikram Nehru Eric Swanson Ashutosh Dubey	April 1993	M. Coleridge- Taylor 33704
WPS1125 Industrial Development and the Environment in Mexico	Adriaan Ten Kate	April 1993	C. Jones 37699
WPS1126 The Costs and Benefits of Slovenian Independence	Milan Cviki Evan Kraft Milan Vodopivec	April 1993	S. Moussa 39019
WPS1127 How International Economic Links Affect East Asia	Vikram Nehru	April 1993	M. Coleridge- Taylor 33704
WPS1128 The International Ocean Transport Industry in Crisis: Assessing the Reasons and Outlook	Hans Jürgen Peters	April 1993	J. Lucas- Walker 31078
WPS1129 How Policy Changes Affected Cocoa Sectors in Sub-Saharan African Countries	Jonathan R. Coleman Takamasa Akiyama Panos N. Varangis	April 1993	G. Ilogon 33732
WPS1130 Poverty and Policy	Michael Lipton Martin Ravallion	April 1993	P. Cook 33902
WPS1131 Prices and Protocols in Public Health Care	Jeffrey S. Hammer	April 1993	J. S. Yang 81418
WPS1132 An Analysis of Repressed Inflation in Three Transitional Economies	Andrew Feltenstein Jiming Ha	April 1993	E. Zamora 33706
WPS1133 Macroeconomic Framework for an Oil-Based Economy: The Case of Bahrain	Ibrahim Elbadawi Nader Majd	April 1993	A. Maranon 31450
WPS1134 Managing a Nonrenewable Resource: Savings and Exchange-Rate Policies in Bahrain	Ibrahim A. Elbadawi Nader Majd	April 1993	A. Maranon 31450
WPS1135 Inflation in Czechoslovakia, 1985-91	Zdenek Drabek Kamil Janacek Zdenek Tuma	May 1993	E. Zamora 33706
WPS1136 The Dynamic Behavior of Quota License Prices: Theory and Evidence from the Hong Kong Apparel Quotas	Kala Krishna Ling Hui Tan	May 1993	D. Gustafson 33714

# Policy Research Working Paper Series

Title	Author	Date	Contact for paper
WPS1137 Railway Reform in the Central and Eastern European Economies	Philip W. Blackshaw Louis S. Thompson	May 1993	TWJTD 31005
WPS1138 The Economic Impact of Military Expenditures	Daniel Landau	May 1993	C. Jones 37699
WPS1139 Should Sub-Saharan Africa Expand Cotton Exports?	Jonathan R. Coleman M. Elton Thigpen	May 1993	G. Ilogon 33732
WPS1140 How Retail Food Markets Responded to Price Liberalization in Russia after January 1992	Bruce Gardner Karen M. Brooks	May 1993	C. Spooner 32116
WPS1141 Foreign Direct Investment in a Macroeconomic Framework: Finance, Efficiency, Incentives, and Distortions	Maxwell J. Fry	May 1993	R. Vo 31047
WPS1142 Rent-Seeking Trade Policy: A Time-Series Approach	Martin Rama	May 1993	D. Ballantyne 37947
WPS1143 Tariff Rates, Tariff Revenue, and Tariff Reform: Some New Facts	Lant Pritchett Geeta Sethi	May 1993	M. Fernandez 33766
WPS1144 The Foreign Trade Dimension of the Market Transition in Poland: The Surprising Export Performance and Its Sustainability	Bartlomiej Kaminski	June 1993	P. Kokila 33716
WPS1145 The Simple(r) Algebra of Pension Plans	Dimitri Vittas	June 1993	P. Infante 37664
WPS1146 Is Poverty Increasing in the Developing World?	Shaohua Chen Gaurav Datt Martin Ravallion	June 1993	P. Cook 33902
WPS1147 Interest Rates, Growth, and External Debt: The Macroeconomic Impact of Mexico's Brady Deal	Stijn Claessens Daniel Oks Sweder van Wijnbergen	June 1993	R. Vo 33722